Chapter 17
Capacitors Chapter Review

EQUATIONS:

- $C = \frac{q}{V_C}$ [This is the definition of capacitance. Specifically, the capacitance of a capacitor is the RATIO of the charge q on one capacitor plate to the voltage V_C across the plates. This relationship is ALWAYS true.]

- $C_{eq} = C_1 + C_2 + C_3 + \ldots$ [The equivalent capacitance of a parallel combination of capacitors equals the sum of the individual capacitances in the combination. As such, the equivalent capacitance of a parallel combination will be larger than any of the elements in the combination. Note that this expression has the same form as the equivalent resistance of a series combination of resistors.]

- $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$ [The inverse of the equivalent capacitance of a series combination of capacitors equals the sum of the inverse of each of the individual capacitors in the combination. As such, the equivalent capacitance of a series combination will be smaller than the smallest of the elements in the combination. Note that this expression has the same form as the equivalent resistance of a parallel combination of resistors.]

- $i = \frac{dq}{dt}$ [This is the formal definition of current, where dq is the differential amount of charge that passes by a point within a circuit during the differential time interval dt. This is useful as there will be times when you will want to relate the charge q accumulated on a capacitor plate to the voltage $V_C = \frac{q}{C}$ across the plates and the current $i = \frac{dq}{dt}$ flowing in the branch.]

- $i = i_o e^{-\frac{t}{RC}}$ [This relationship, derived from the solution of Kirchoff's Laws, describes the current in a single loop RC circuit as the circuit's capacitor charges up. The accumulation of charge follows the relationship $q(t) = q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$, though you will use this expression rarely, if ever.]

- $\tau = RC$ [This is the definition of one time constant for an RC circuit. The time constant is the amount of time it takes an initially uncharged capacitor in an RC circuit to charge up to 63% of its maximum value. In that time, the current will also drop to 37% of its maximum.]

- $C = \frac{q}{V_C}$, $\Delta V = -\int \mathbf{E} \bullet d\mathbf{r}$, $V_C = -\Delta V$, $\kappa \int \mathbf{E} \bullet d\mathbf{S} = \frac{q_{encl}}{\varepsilon_o}$ [In order, these expressions are: the definition of capacitance, the relationship between an electric field and the voltage difference between two points in the electric field, the relationship between the capacitor's defined voltage V_C and the voltage difference between the plates of a capacitor (assuming you are moving in the direction of the electric field between the plates--that is, from the high voltage plate to the low voltage plate), and Gauss's Law (where k is the dielectric constant for the medium between the plates). Why are they all in one place? Each is a component in the technique required to derive a capacitance expression from scratch in terms of the cap's physical parameters (i.e., its plate area, the distance between its plates, etc.).]

- $C_{\text{with dielectric}} = kC_{\text{w/o dielectric}}$ [This defines the dielectric constant.]

- $W = \frac{1}{2} C V_C^{\,2}$ [This is the amount of work required to charge a capacitor of capacitance C up to a voltage V_C. More importantly, this is the amount of energy stored in a charged capacitor.]

- mf, μf, nf, pf [The MKS unit for capacitance is farads. One farad is a very large capacitance. In most cases, a capacitor's value will be in the milli, micro, nano, or pico range. These are denoted as: millifarads (10^{-3} farads), microfarads (10^{-6} farads), nanofarads (10^{-9} farads), and picofarads (10^{-12} farads). Although these abbreviations are convenient, be aware. Whenever you do a problem in which a capacitance value is required, the value must be expressed in farads. That means you must be able to convert, say, 3.2 pf to $3.2\text{x}10^{-12}$ farads. If you don't, you will find yourself off by a factor of a trillion . . .]

COMMENTS, HINTS, and THINGS to be aware of:

- Capacitors in DC circuits are used to store energy. They do this when a certain amount of negative charge is placed on one plate while an equal amount of opposite (positive) charge is placed on the other plate. This sets up an electric field between the plates. The energy is stored in the electric field.

- Charge doesn't really flow through a capacitor. It flows onto one plate, electrostatically repulsing an equal amount of like charge off the second plate (this leaves that second plate oppositely charged).

- Because the amount of charge placed on one plate during a differential time interval is equal to the amount of charge electrostatically repulsed off the second plate during that same interval, the current on either side of a capacitor will be the same. That is, at any instant, current in a branch in which there exists a capacitor will, as always, be the same everywhere within the branch.

- The charging characteristic of a capacitor mirrors the discharging characteristic of a capacitor. That is, if it takes 2 seconds to charge a capacitor to 60% of its maximum, it will take 2 seconds to discharge 60% of that same capacitor's accumulated charge (this assumes that the circuit's resistor is the same in both cases).

- Capacitors in parallel have a common voltage but different amounts of charge on their plates.

- Capacitors in series have a common amount of charge on their plates but different amounts of voltage across their plates.

- Don't be put off if you are asked to execute Kirchoff's Laws for a circuit in which capacitors exist. Just as the voltage across a resistor is iR, the voltage across a capacitor is $\frac{q}{C}$. Just as a resistor in a circuit will have a high voltage and low voltage side, a capacitor will also have a high voltage and low voltage side. So, if asked to write out Kirchoff's Laws for a complex circuit that includes capacitors, do as usual. Define a current for each branch, mentally noting that the current in a capacitor's branch will be time varying (again, no big deal--assume you are dealing with a specific point in time and call the current at that time $i_{whatever}$). Sum the voltage changes around closed loops, as usual, noting that the only additional information needed for capacitor branches is a relationship between the capacitor's charge q and the branch's current i . As that relationship is nicely summarized by $i_{whatever} = \frac{dq}{dt}$, it's no big deal. Note, lastly, that although you most probably won't be asked to actually solve anything beyond the most elementary of situations, you'd better not forget the $i_{whatever} = \frac{dq}{dt}$ relationship(s) when writing out the governing expressions for the circuit. Without it (them), a solution would be impossible to produce.

- Don't be put off by dielectrics. The idea of a dielectric constant was devised to allow you to ignore the induced charge on a dielectric surface placed between the plates of a charged capacitor. That is, when using Gauss's Law to determine the electric field between the plates (you might need to do this so that you can determine the voltage difference across the plates and, hence, the capacitance q/V_C specific to the particular geometry incorporated into your capacitor), the amount of charge inside the Gaussian surface will be the charge on the plate (inside the G.S.) added to the charge induced on the dielectric surface. Multiplying by the dielectric constant allows you to ignore this latter charge and still get a meaningful electric field expression.